lecture 5. particle interactions and electromagnetic processes

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Advanced Astroparticle Physics

NPAC M2

2024-2025

in today's class...

- **relativistic kinematics**
  - review of relativistic kinematics
- **cross sections**
- **particle interactions**
  - general formulation
  - mean free paths
- **propagation of cosmic particles**
  - electrons
  - photons
  - nuclei
  - neutrinos

relativistic kinematics

review. phase space and amplitudes

generic scattering process

$$X_1 + \ldots + X_{n_i} \to X_{n_i+1} + \ldots + X_{n_i+n_f}$$

n_i = 1 or 2

scattering perturbation theory in momentum-space basis

$$\langle \vec{p}_1, \ldots, \vec{p}_{n_i} \,|\, S \,|\, \vec{p}_{n_i+1}, \ldots, \vec{p}_{n_i+n_f} \rangle = \mathbb{1} - i(2\pi\hbar)\, \delta^4\left(\sum_{j=1}^{n_i} P_j - \sum_{j=n_i+1}^{n_i+n_f} P_j \right) \frac{\mathscr{M}(\vec{p}_1, \ldots, \vec{p}_{n_i}; \vec{p}_{n_i+1}, \ldots, \vec{p}_{n_i+n_f})}{\prod_{j=1}^{n_i+n_f} \sqrt{2E_j}}$$

S-matrix

(from perturbation theory)

four-momentum conservation

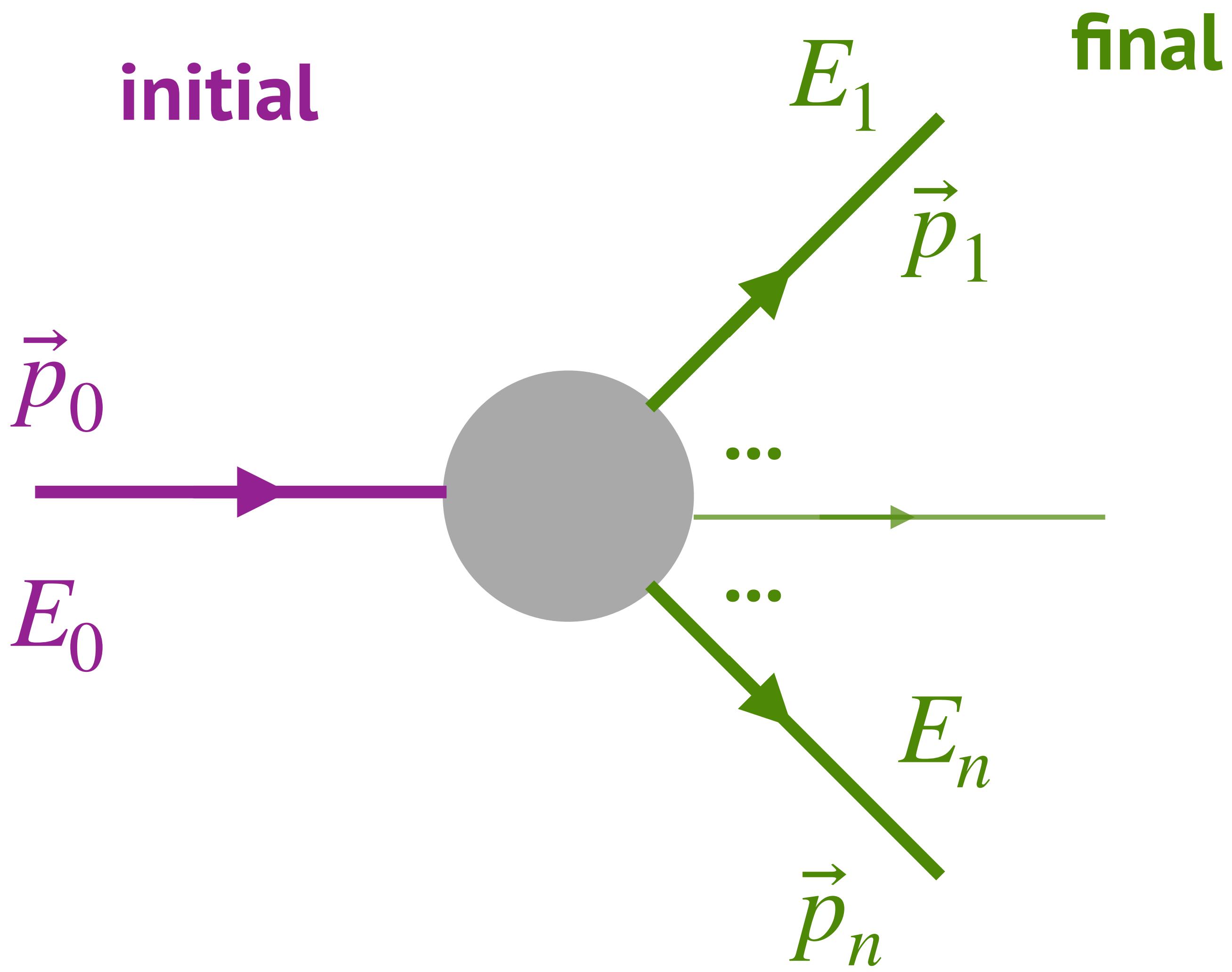
"amplitudes"

$|M|^2$

Lorentz-invariant phase space

$$d\Pi_{ps} = (2\pi\hbar)^4\, \delta^4\left(\sum_{j=1}^{n_i} P_j - \sum_{j=n_i+1}^{n_i+n_f} P_j \right) \prod_{j=n_i+1}^{n_i+n_f} \frac{1}{2E_j} \frac{d^3 p_j}{(2\pi\hbar)^3}$$

$$X_0 \to X_1 + \ldots + X_n$$



decay rate

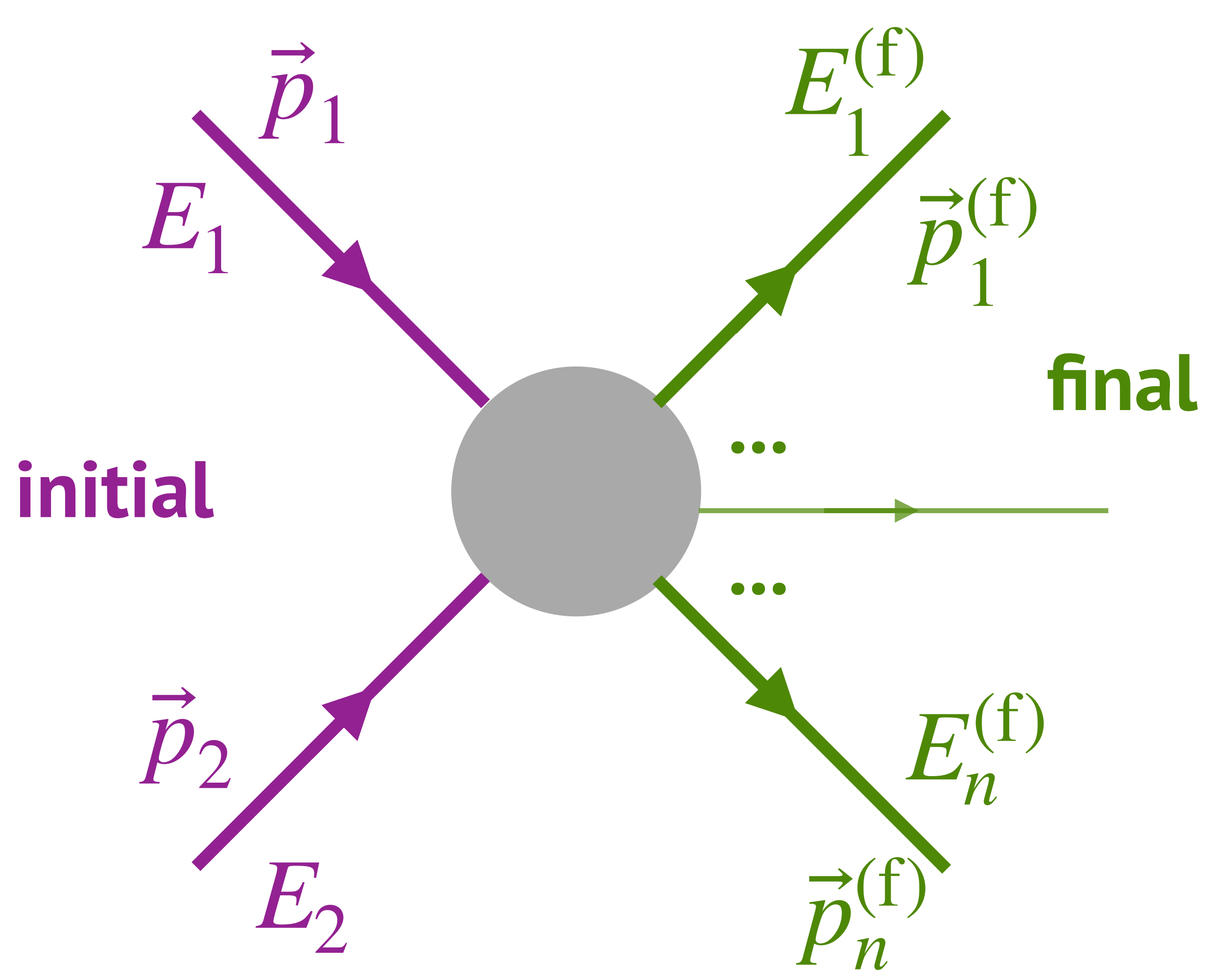
$$d\Gamma = \frac{|\mathscr{M}|^2}{2m_0c^2} d\Pi_{ps}(P_0; P_1, \ldots, P_n)$$

$$d\Gamma = \frac{|\mathscr{M}|^2}{2m_0c^2}(2\pi\hbar)^4\delta^4\left(P_0 - \sum_{j=1}^{n} P_j\right)\prod_{j=1}^{n}\frac{1}{2E_j}\frac{d^3p_j}{(2\pi\hbar)^3}$$

lifetime

$$\tau = \Gamma^{-1}$$

$$X_1 + \ldots + X_{n_\mathrm{i}} \to X_{n_\mathrm{i}+1} + \ldots + X_{n_\mathrm{i}+n_\mathrm{f}}$$

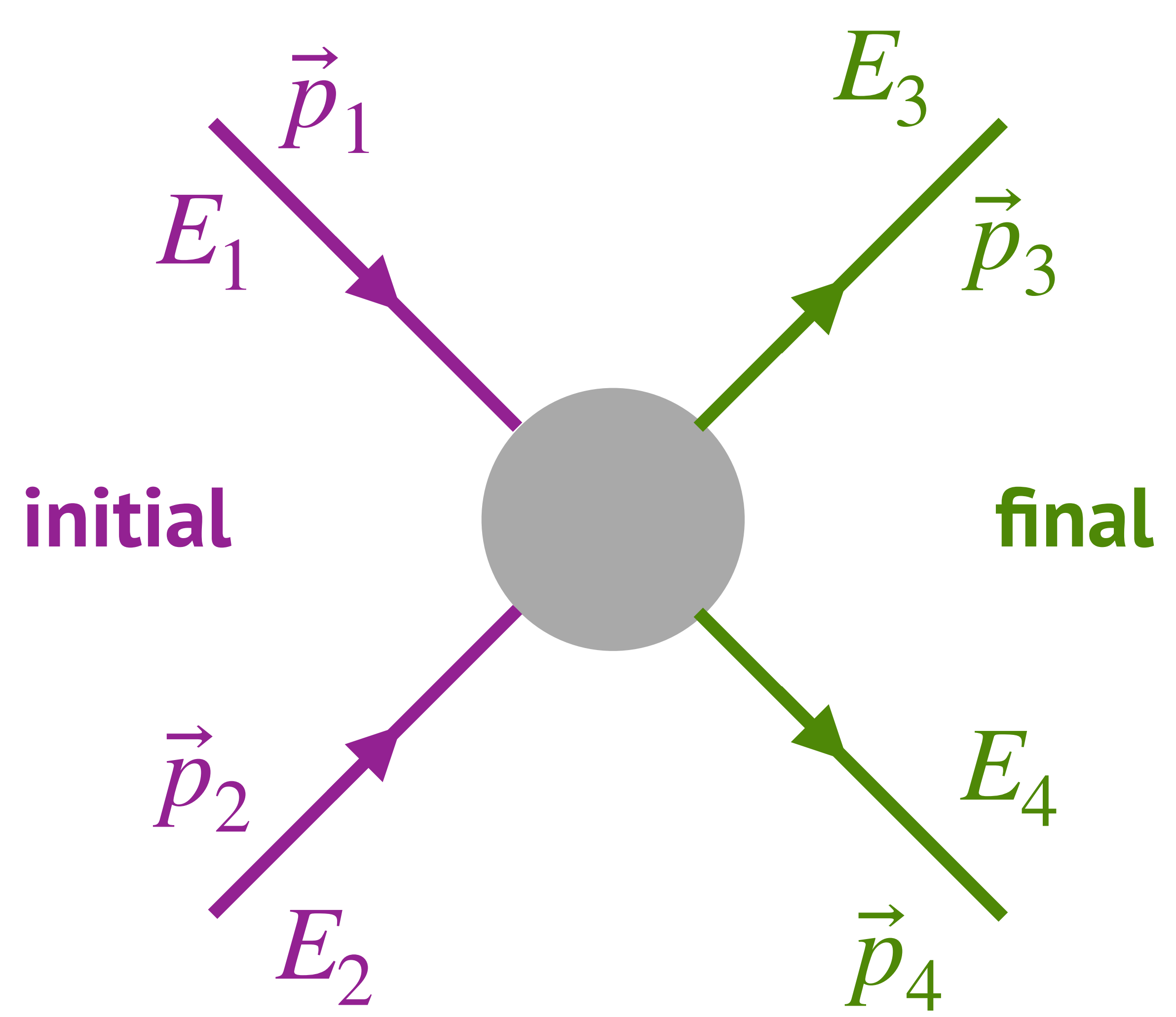


differential cross section

$$\mathrm{d}\sigma = \frac{|\mathscr{M}|^2}{4\sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2 c^4}} \mathrm{d}\Pi_{\mathrm{ps}}(P_1, P_2;\ P_3, \ldots, P_{2+n})$$

$$\mathrm{d}\sigma = \frac{|\mathscr{M}|^2}{4\sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2 c^4}} (2\pi\hbar)^4 \delta^4\left(P_1 + P_2 - \sum_{j=3}^{2+n} P_j\right) \prod_{j=1}^{n} \frac{1}{2E_j} \frac{\mathrm{d}^3 p_j}{(2\pi\hbar)^3}$$

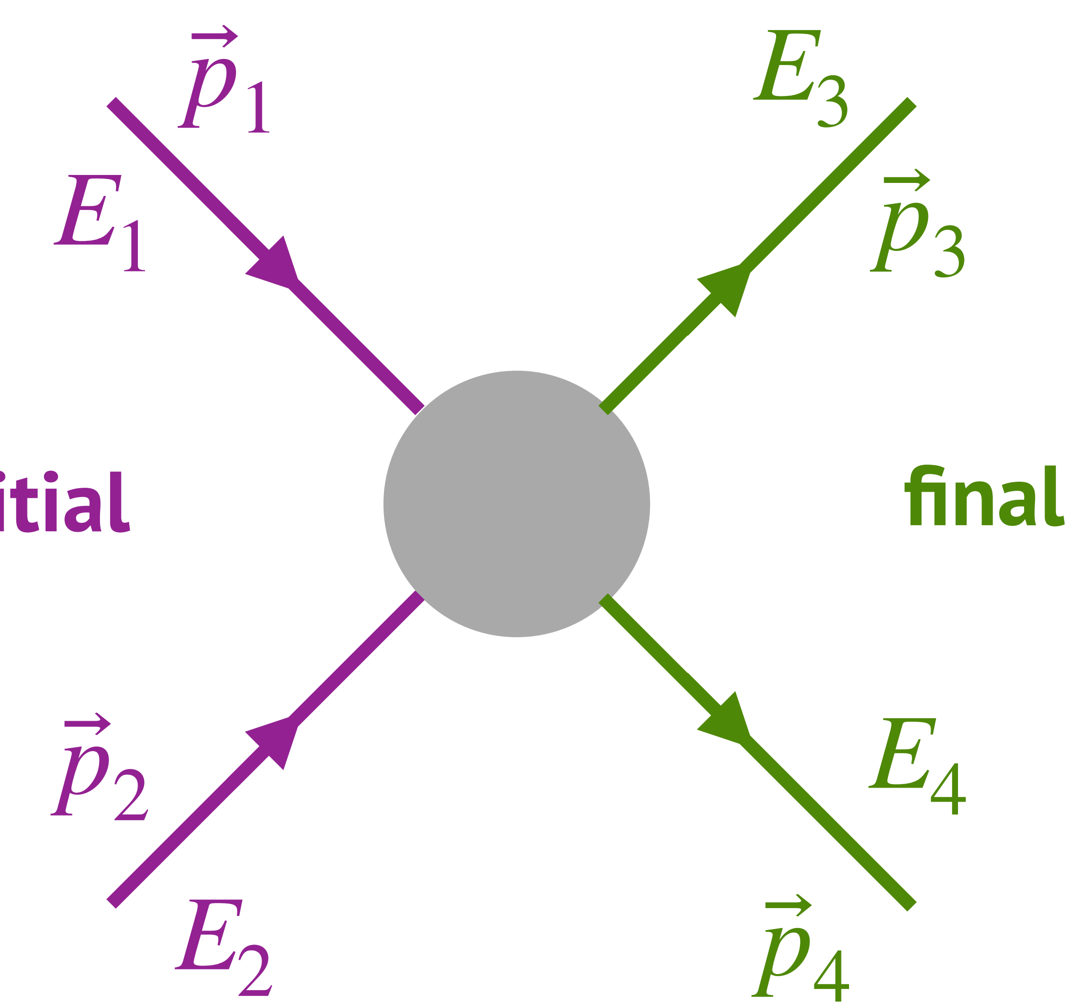
$$X_1 + X_2 \to X_3 + X_4$$



differential cross section

$$\mathrm{d}\sigma = \frac{|\mathscr{M}|^2}{4\sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2 c^4}} \mathrm{d}\Pi_{\mathrm{ps}}(P_1, P_2;\ P_3, P_4)$$

$$\mathrm{d}\sigma = \frac{|\mathscr{M}|^2 (2\pi\hbar)^4 \delta^4\left(P_1 + P_2 - P_3 - P_4\right)}{4\sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2 c^4}} \frac{1}{2E_3} \frac{\mathrm{d}^3 p_3}{(2\pi\hbar)^3} \frac{1}{2E_4} \frac{\mathrm{d}^3 p_4}{(2\pi\hbar)^3}$$



Mandelstam variables

$$s = (P_1 + P_2)^2 = (P_3 + P_4)^2 = m_1^2c^4 + m_2^2c^4 + 2E_1E_2(1 - \vec{\beta}_1 \cdot \vec{\beta}_2)$$

$$t = (P_1 - P_3)^2 = (P_2 - P_4)^2 = m_1^2c^4 + m_3^2c^4 - 2E_1E_3(1 - \vec{\beta}_1 \cdot \vec{\beta}_3)$$

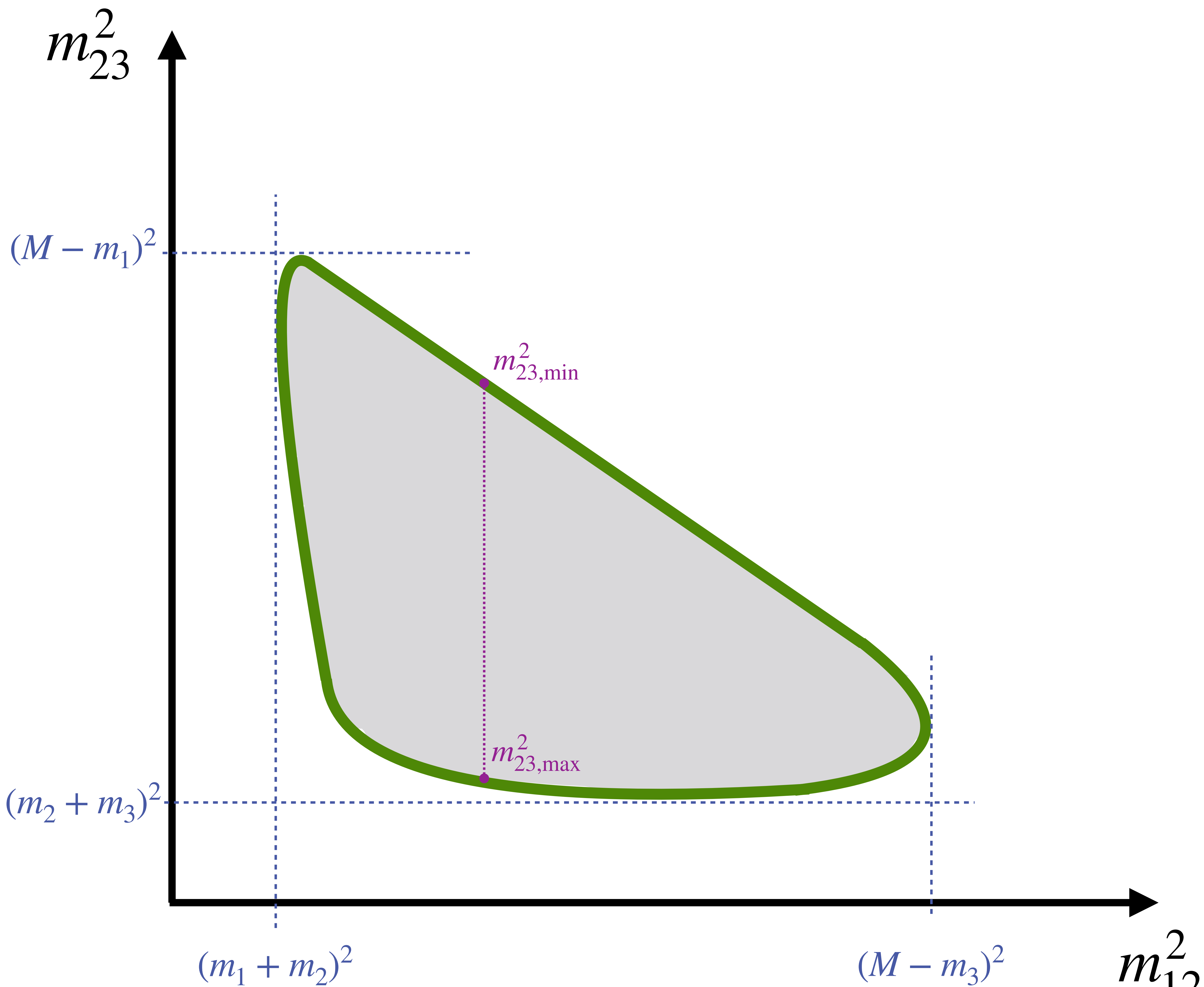
$$u = (P_1 - P_4)^2 = (P_2 - P_3)^2 = m_1^2c^4 + m_4^2c^4 - 2E_1E_4(1 - \vec{\beta}_1 \cdot \vec{\beta}_4)$$

centre of mass energy (squared)

$$s = m_1^2c^4 + m_2^2c^4 + 2E_1E_2(1 - \beta_1\beta_2 \cos\theta)$$

relative velocity

$$\beta_{\text{rel}} = \sqrt{\frac{(P_1 \cdot P_2)^2 - (m_1m_2c^2)^2}{(P_1 \cdot P_2)^2}}$$



$$m^2_{23,\mathrm{min}}c^4 = \left(E_2^\star + E_3^\star\right)^2 - \left(\sqrt{E_2^{\star 2} - m_2c^4} + \sqrt{E_3^{\star 2} - m_3c^4}\right)^2$$

$$m^2_{23,\mathrm{max}}c^4 = \left(E_2^\star + E_3^\star\right)^2 - \left(\sqrt{E_2^{\star 2} - m_2c^4} - \sqrt{E_3^{\star 2} - m_3c^4}\right)^2$$

$$E_2^\star = \frac{m_{12}^2 - m_1^2 + m_2^2}{2m_{12}}c^2 \qquad E_3^\star = \frac{M^2 - m_{12}^2 - m_2^2}{2m_{12}}c^2$$

- suppose decay of particle of mass M into 3 particles
- the star indicates the frame wherein the combined '12' particle is at rest
- also applicable to scatterings

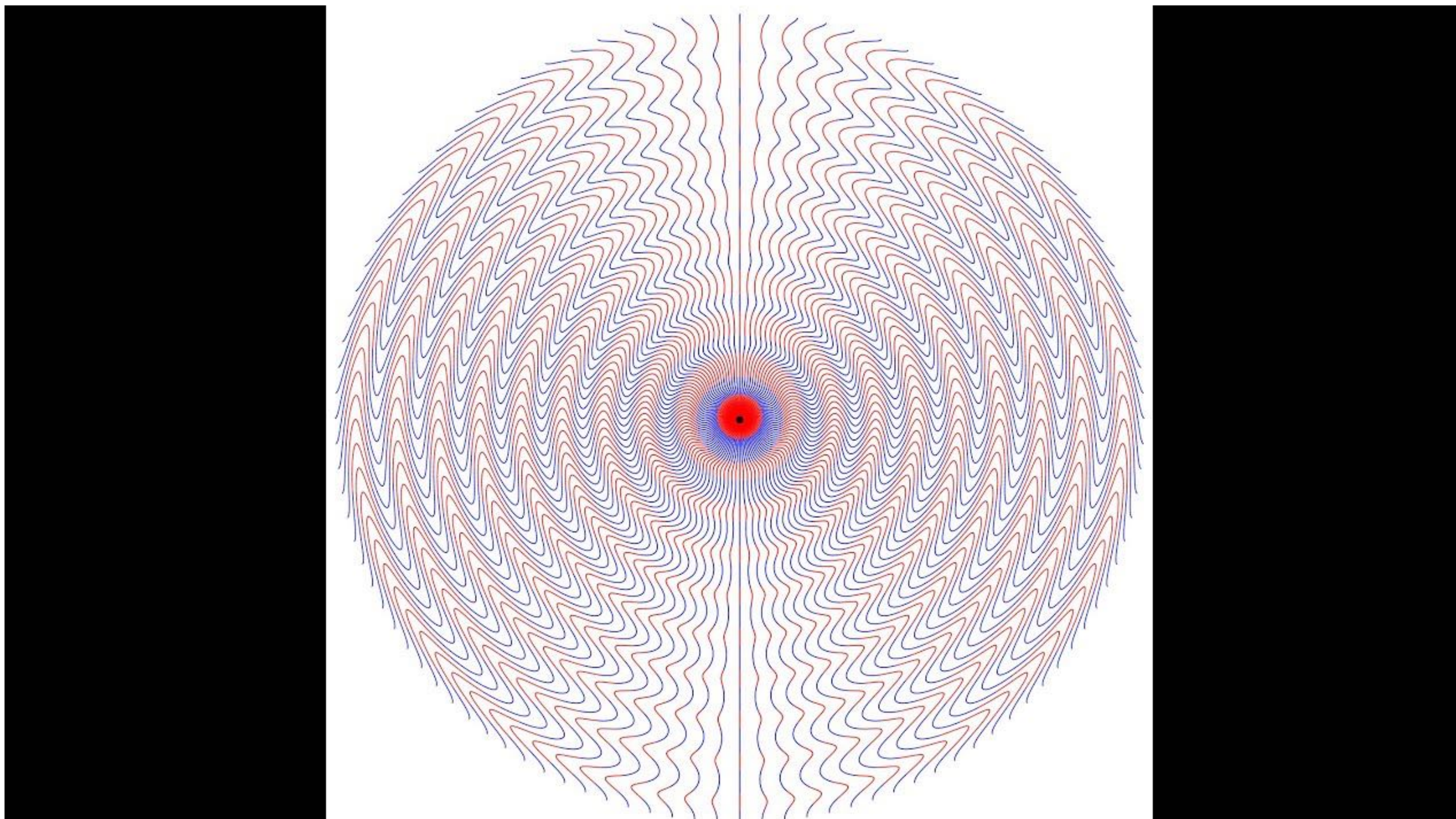
- **total cross section:** $\sigma_{\mathrm{tot}} = \sigma_{\mathrm{ela}} + \sigma_{\mathrm{ine}}$
  - elastic: A + B → A + B
  - inelastic: A + B → C + D + ...
- useful to work in the lab frame: $\sigma_{\mathrm{p+t\to f}} = \frac{1}{\Phi_i}\frac{\mathrm{d}N_f}{\mathrm{d}t}$
- inclusive cross section: $\frac{\mathrm{d}^3\sigma_{\mathrm{p+t\to f}}}{\mathrm{d}^3 p_{\mathrm{f}}} = \frac{1}{\Phi_{\mathrm{p}}}\frac{\mathrm{d}^4 N_f}{\mathrm{d}^3 p_{\mathrm{f}}\,\mathrm{d}t}$
- exclusive cross section: $\frac{\mathrm{d}^3\sigma_{\mathrm{p+t\to f_1+\ldots+f}_n}}{\mathrm{d}^3 p_{\mathrm{f}}} = \frac{1}{\Phi_{\mathrm{p}}}\frac{\mathrm{d}^{3j+1} N_{\mathrm{f}}}{\mathrm{d}^3 p_{\mathrm{f}_1}\,\ldots\,\mathrm{d}^3 p_{\mathrm{f}_n}\,\mathrm{d}t}$

electromagnetic processes

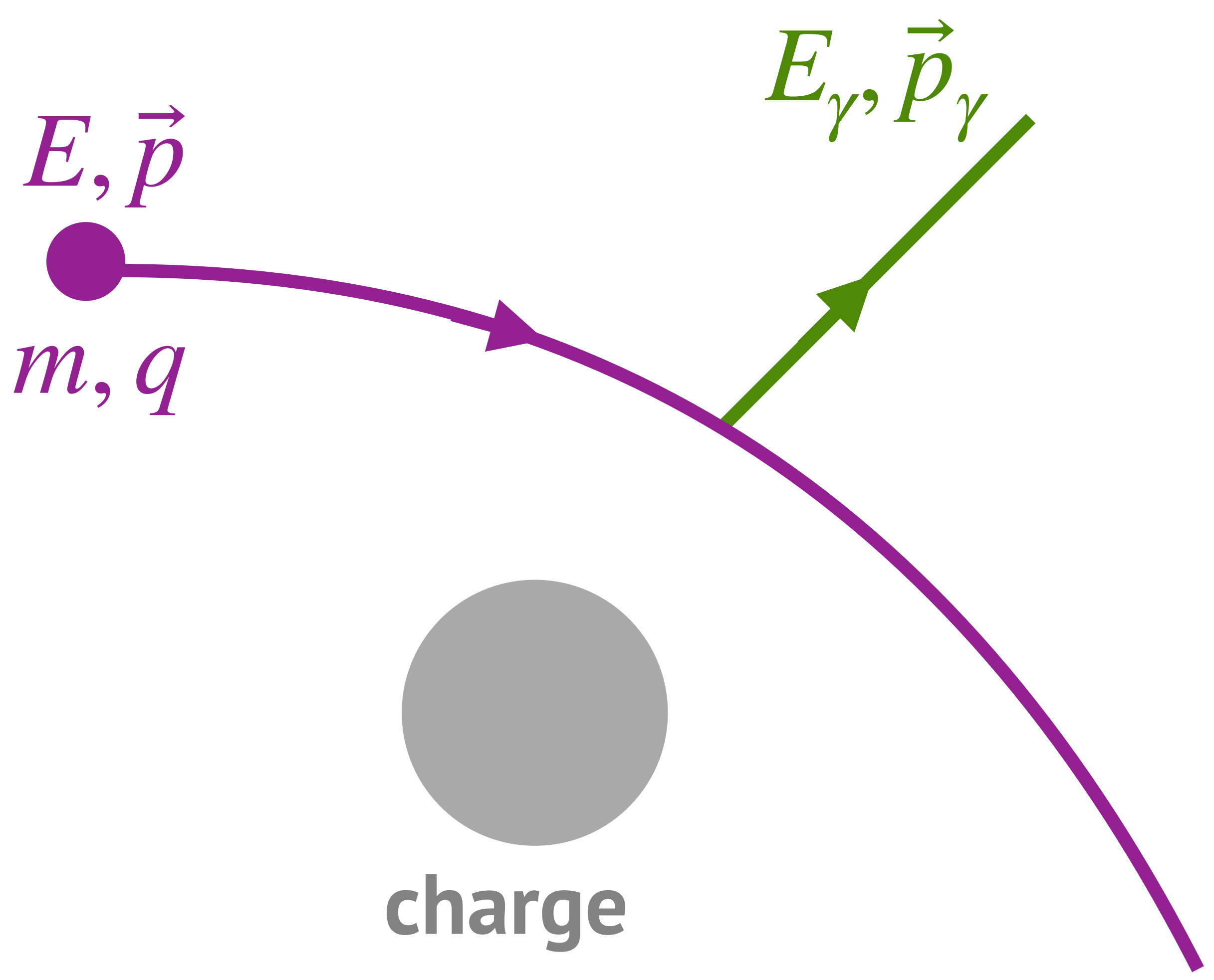
electromagnetic processes: radiation due to an accelerating charge

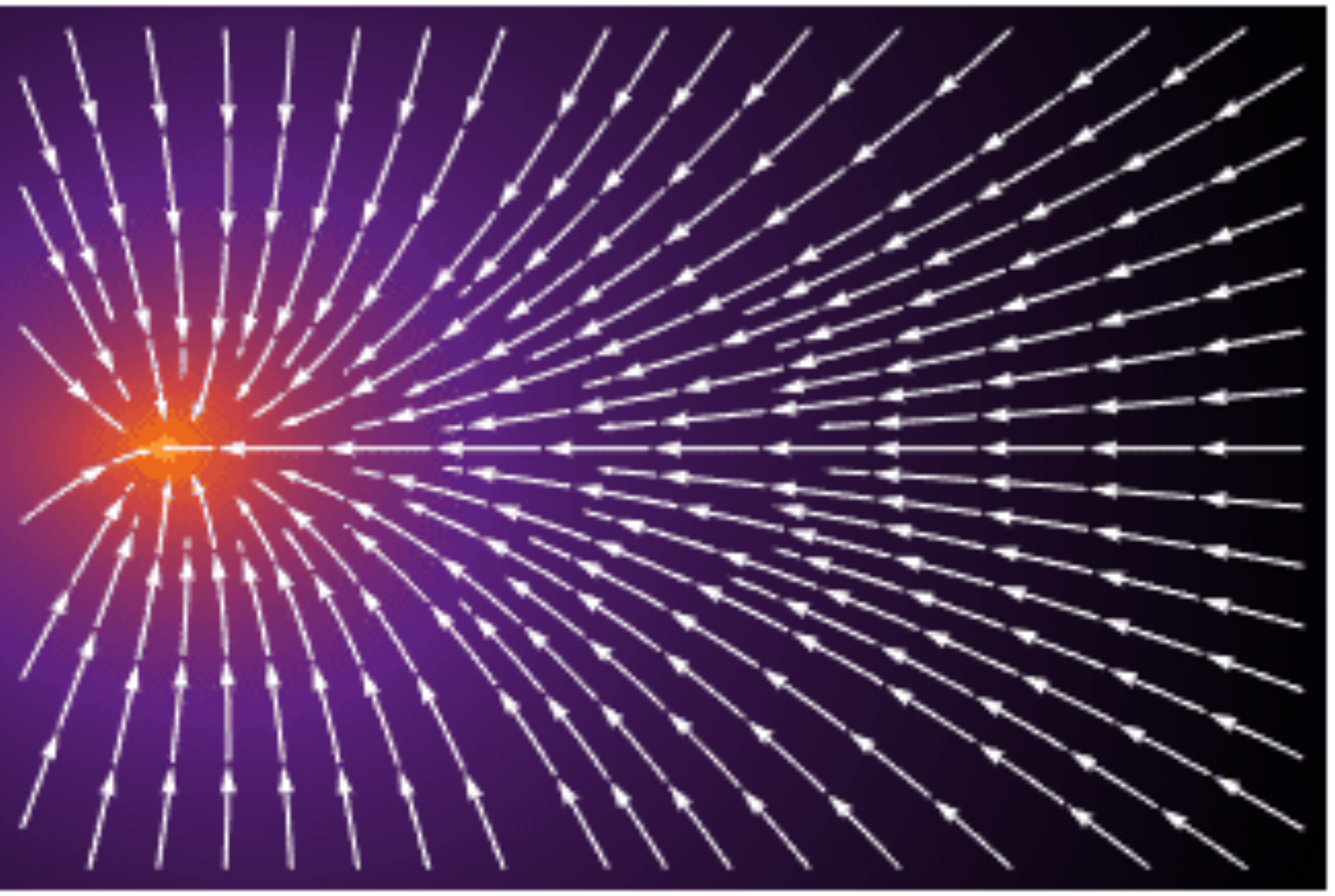
Larmor formula

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{q^2\gamma^4}{6\pi\varepsilon_0 c^3}\left(|a_{\parallel}|^2 + \gamma^2 |a_{\perp}|^2\right)$$

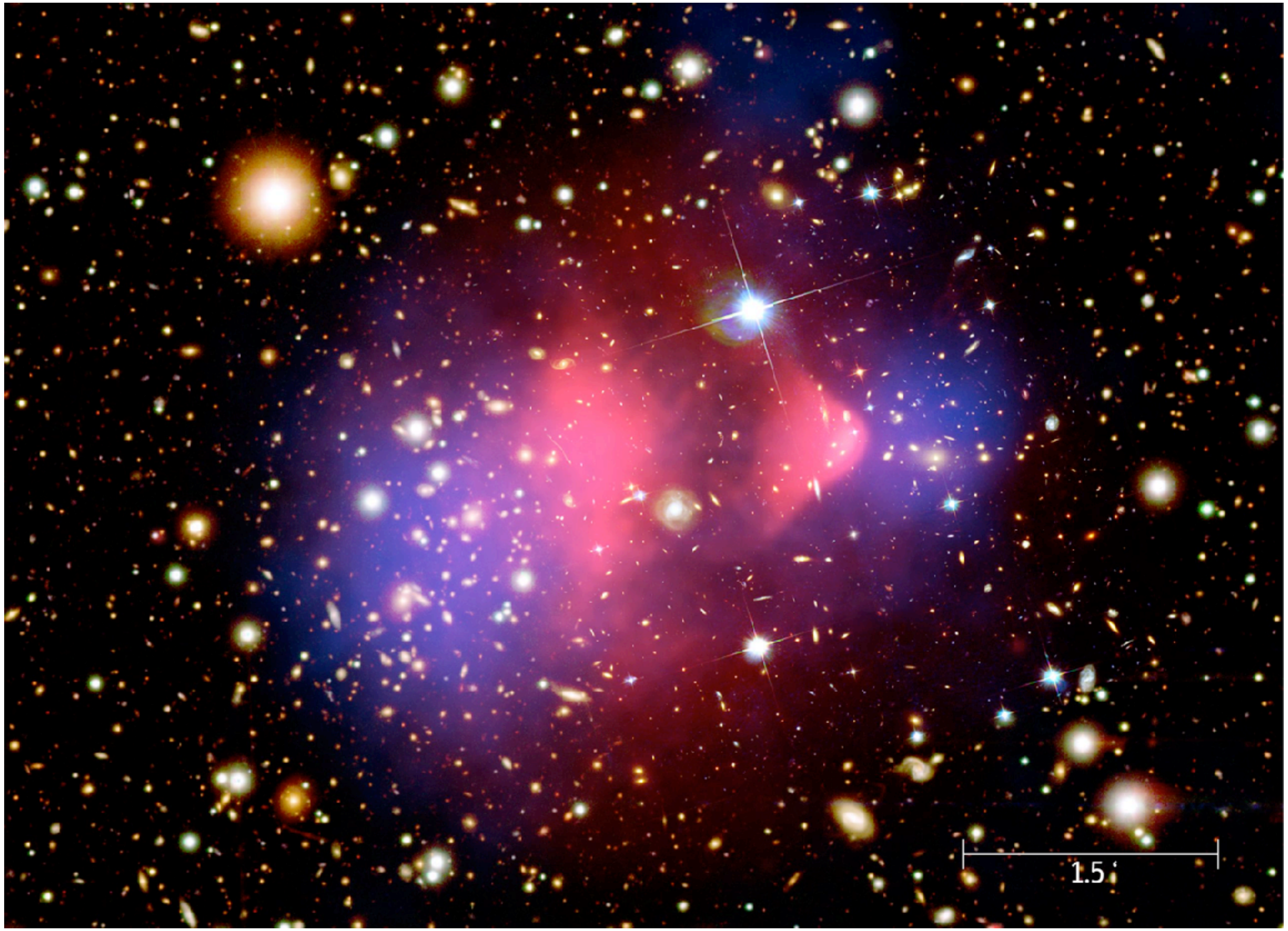


https://www.youtube.com/watch?v=9miwQMxEw_k

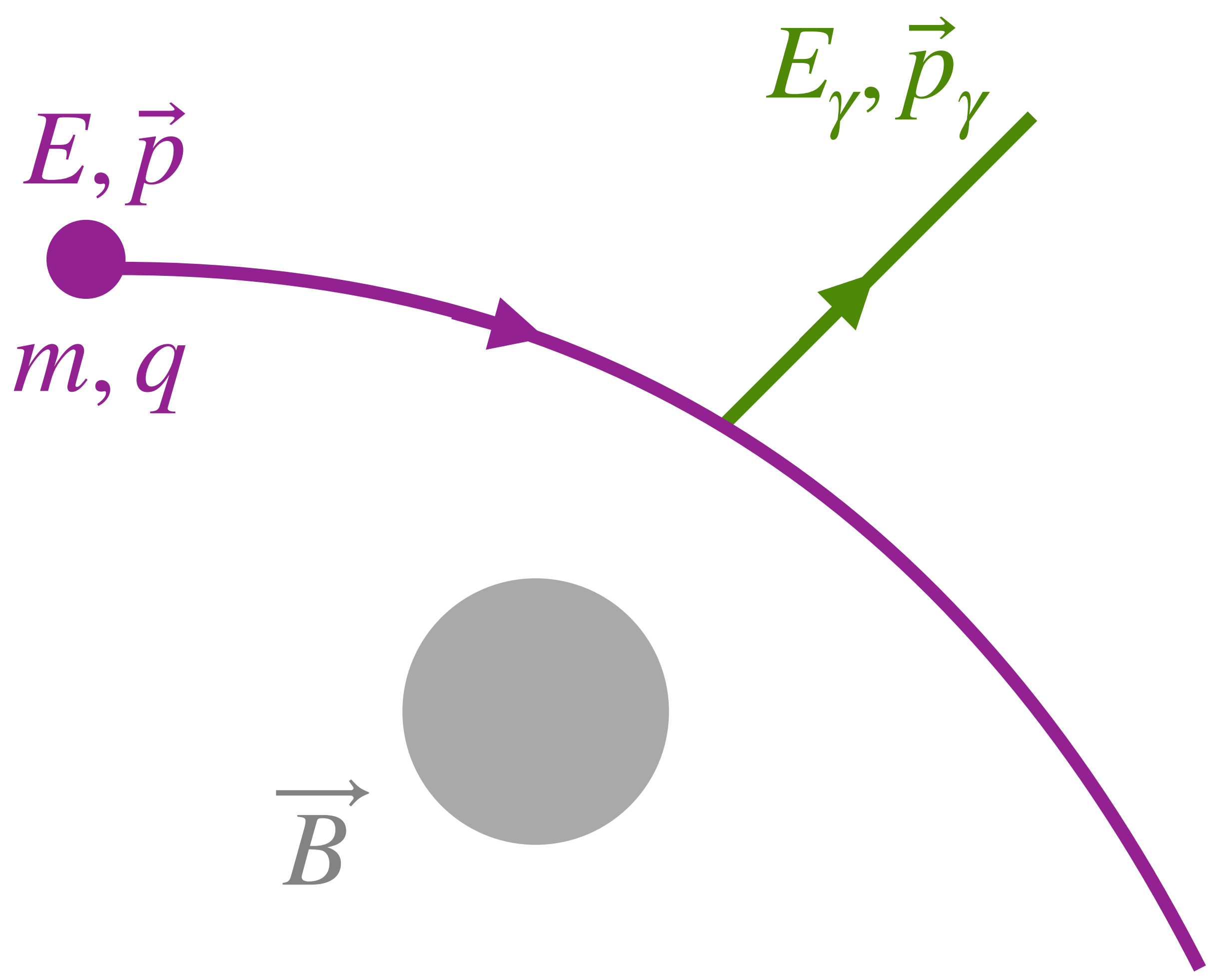




Animations by J. Bertolotti. https://en.wikipedia.org/wiki/Bremsstrahlung#/media/File:Bremsstrahlung.gif



Credits: NASA/CXC/M. Weiss

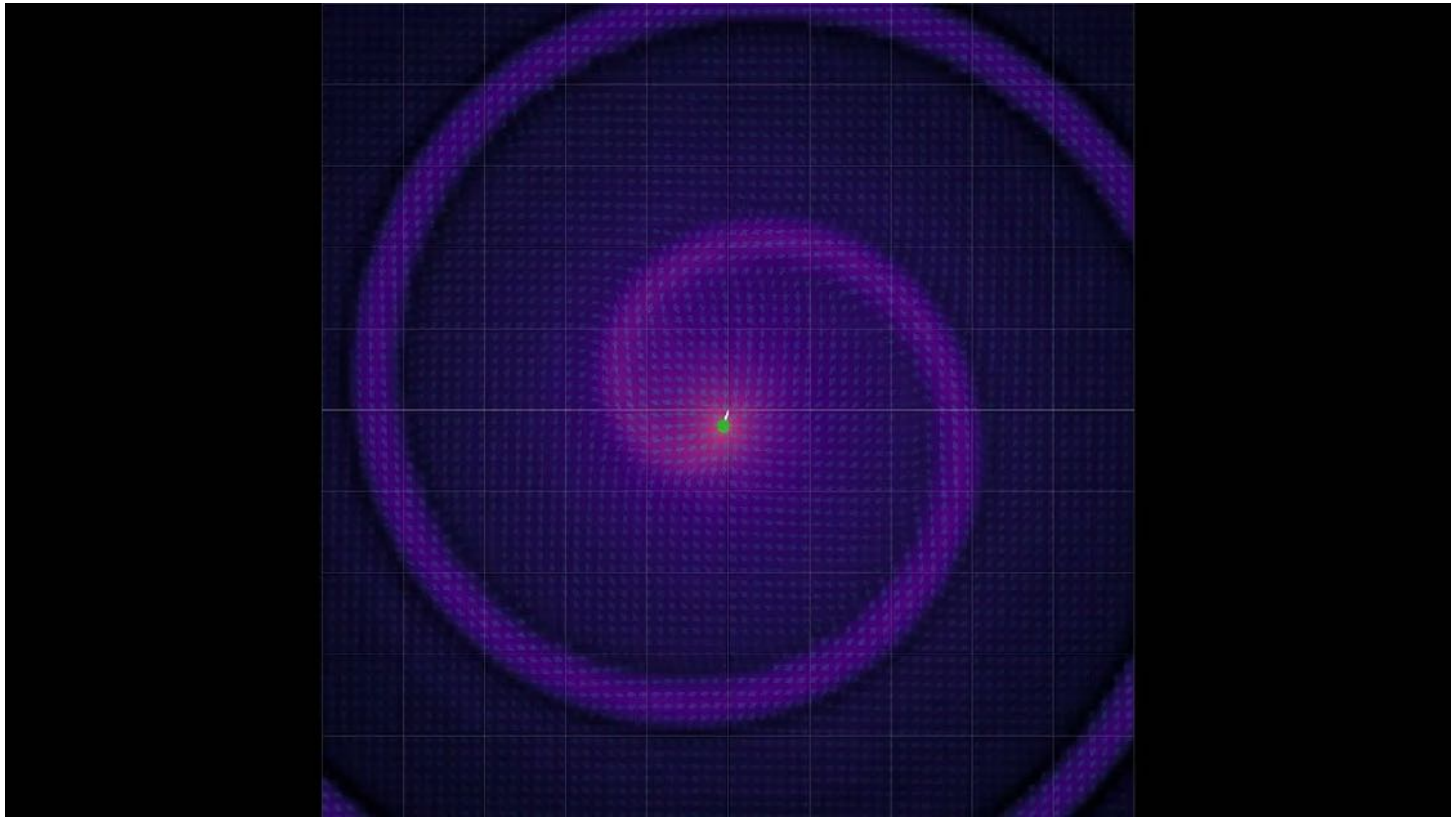


radiated power

$$\frac{\mathrm{d}E}{\mathrm{d}x} = -\frac{q^4}{6\pi\epsilon_0 c^4 m^4}\left|\vec{p}\times\vec{B}\right|$$

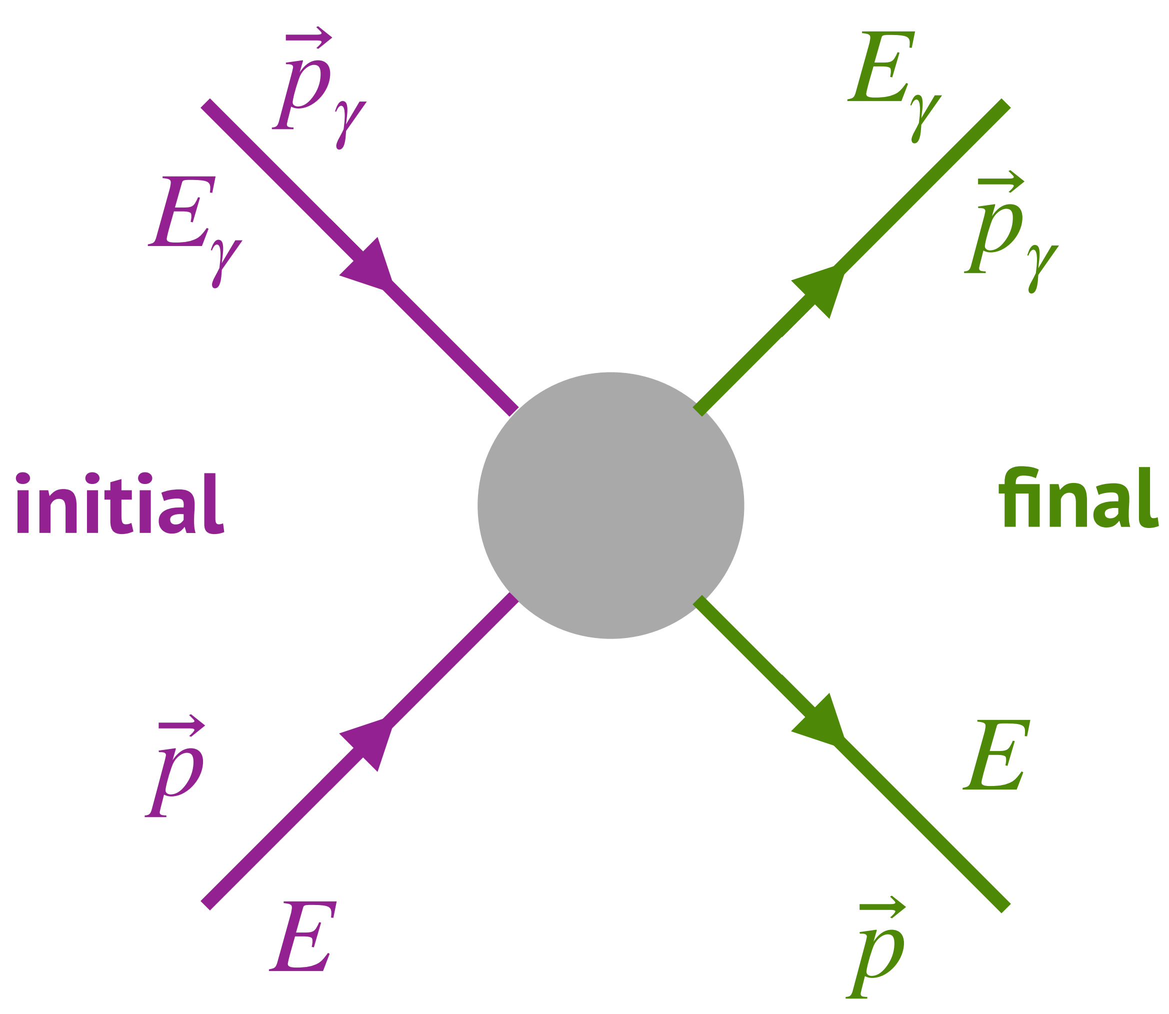
peak energy

$$E_\mathrm{c} = \frac{3}{2}\hbar c\gamma^3 q\frac{1}{p^2}|\vec{B}\times\vec{p}|$$



https://youtube.com/watch?v=VxaObnEudEE

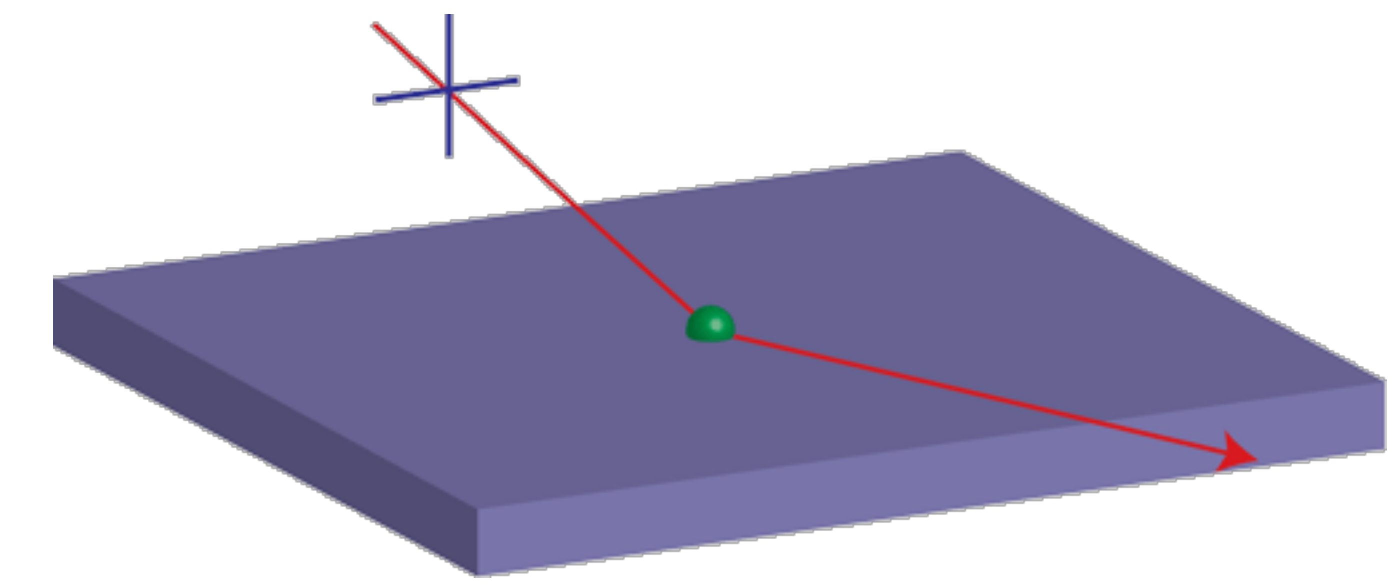
$$q^{\pm} + \gamma \rightarrow q^{\pm} + \gamma$$



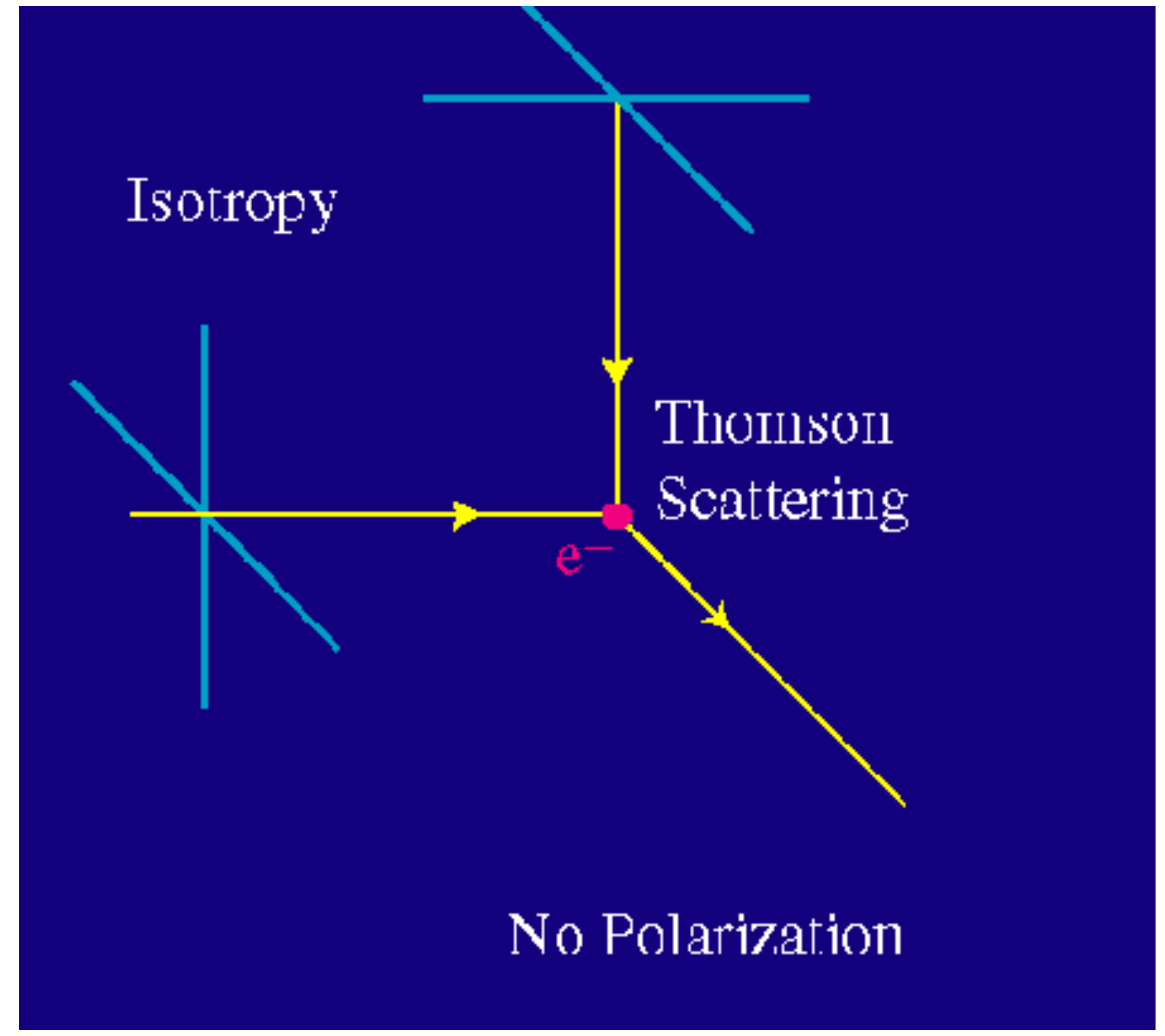
$$\frac{d\sigma_T}{d\Omega} = \frac{q^4}{64\pi^2\varepsilon_0^2 m^2 c^4}(1+\cos^2\theta)$$

$$\sigma_T = \frac{8\pi}{3}\frac{q^4}{(4\pi\varepsilon_0)^2 m^2 c^4}$$

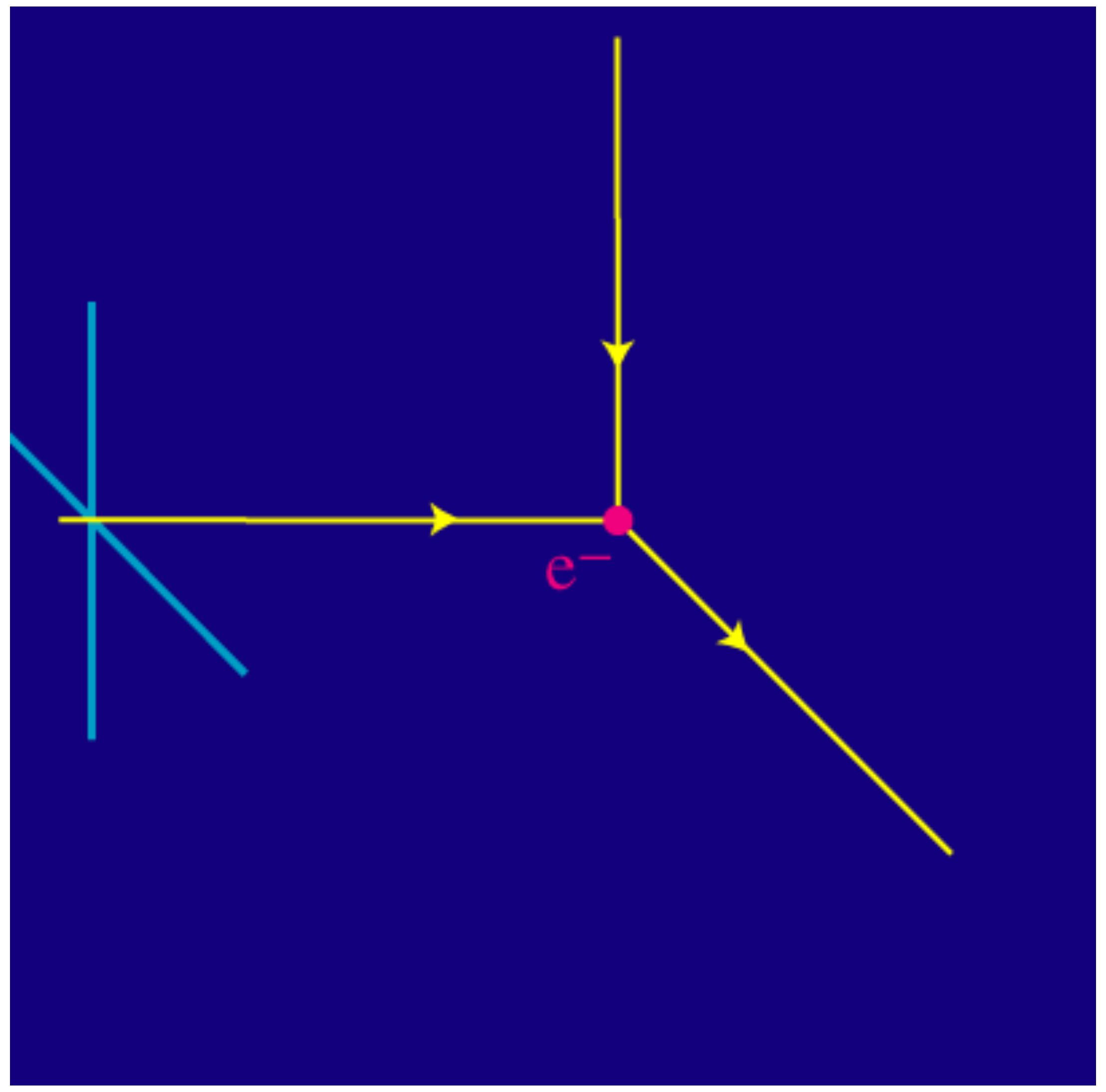
- scattering of charges off photons
- classical limit of Compton scattering
- scattered radiation is *linearly polarised*
- CMB is ~10% polarised due to Thomson scattering



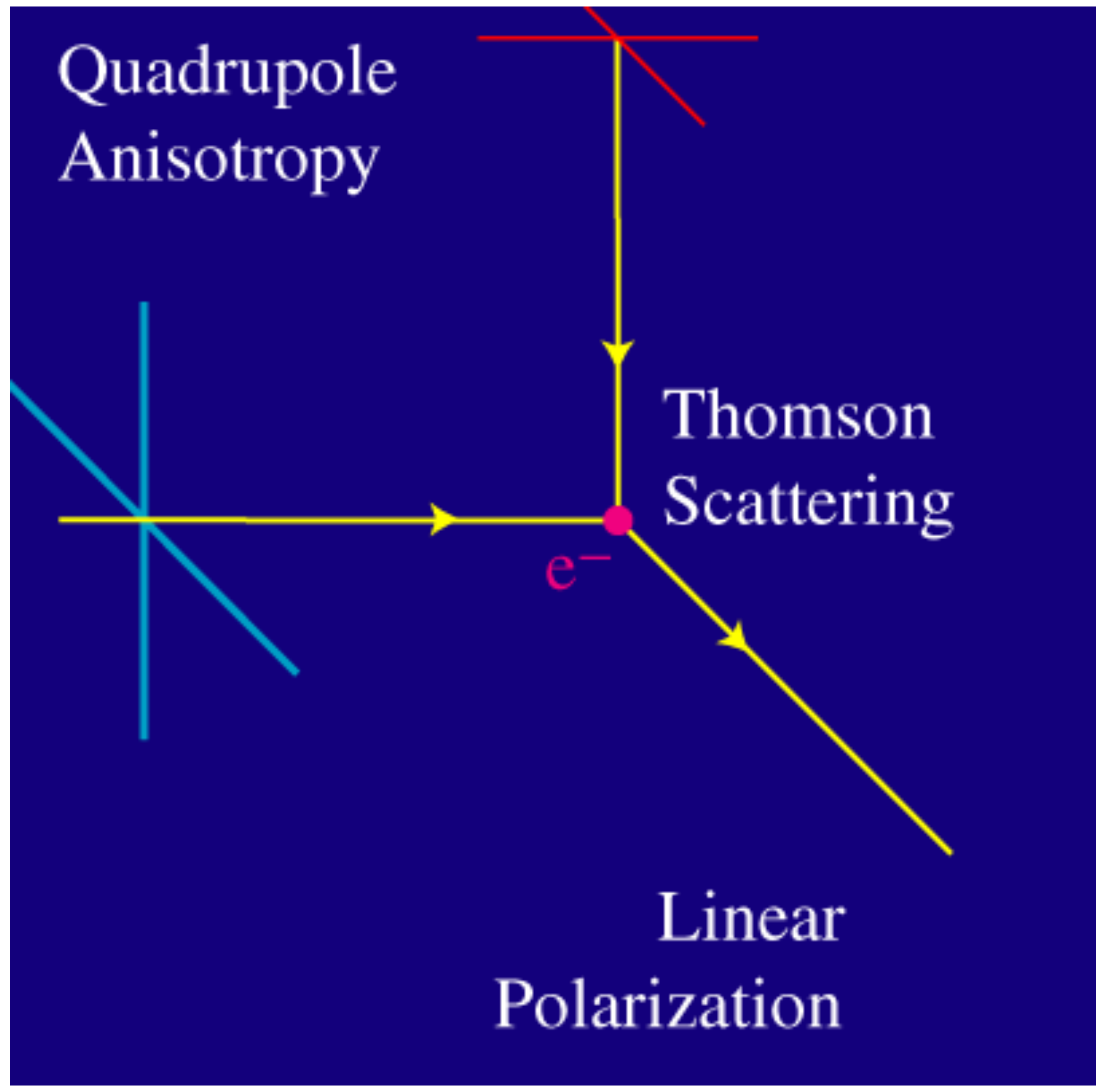
no polarisation



polarised (one state)



polarised (two states)



Animations by W. Hu: https://background.uchicago.edu/~whu/intermediate/Polarization/polar1.html